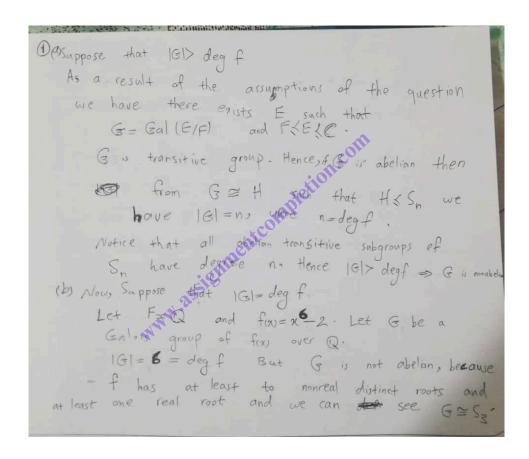
Polynomial Equations and Fields Sample Solution- University of Toronto

- (1) Suppose that F is a subfield of \mathbb{C} , $f(x) \in F[x]$ is irreducible over F and G is the Galois group of f(x) over F.
 - a) Prove that if $|G| > \deg f(x)$, then G is nonabelian.
 - b) Prove or disprove: If $|G| = \deg f(x)$, then G is abelian.
- (2) Suppose that $F \leq E \leq \mathbb{C}$, [E:F] = 100, E is Galois over F and $G = \operatorname{Gal}(E/F)$ contains a subgroup H such that |H| = 25. Use Galois theory to prove that $H \triangleleft G$.
- (3) Let $F \leq \mathbb{C}$. Suppose that $f(x) \in F[x]$ is monic, irreducible over F and deg f(x) = 6. Let E be the splitting field of f(x) over F and let $G = \operatorname{Gal}(E/F)$. Assume that [E:F] = 12 and assume that there exists $\sigma \in G$ such that $|\sigma| = 3$. Let $H = \langle \sigma \rangle$ and $K = E^H$.
 - a) Prove that if $\alpha \in E$ and $f(\alpha) = 0$, then $E = K(\alpha)$.
 - b) Determine how f(x) factors as a product of irreducible polynomals in K[x]. What is the number of irreducible factors of f(x) in K[x] and what are their degrees? (*Hint*: How are $m_{\alpha,K}(x)$ and H related?)



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By the Galois theory for finite Galois groups

We have |G| = [E:F] = 100 cont

|H| = 25 \implies [G:H] = 4 enough

Now by the Sylow theoretis if r be the number of Sylow B-subgroup charder 25 then:

r \ge 1 \pmod{5} enough r \ge 1

r \ge 1 \pmod{5} enough r \ge 1

Sylow r \ge 1

Sylow
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