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Q1 (QID: 4932186). Four people enter a 1,970 kg hatchback and discover that this compresses the hatchback's springs by an additional 0.700 cm. Together the people have a mass of 286 kg.

(a) If we pretend that the hatchback's suspension consists of one spring, then we can calculate its effective spring constant, *k*. What is the value of *k* (in N/m) for this hatchback?

4.00e+05 N/m

(b) The four people exit the hatchback. They push down upon it once and then release it. This causes the hatchback to undergo simple harmonic motion. What is the frequency (in Hz) of this vibrating motion?

2.27 Hz

SOL:

(a) From the fact that addition load of four people of *m* = 286 kg results in compression of the spring by 0.700 cm ($\Delta x$), the effective spring constant can be obtained from Hooke's law:

$$k = \frac{F}{\Delta x} = \frac{mg}{\Delta x} = \frac{(286\ \text{kg})(9.80\ \text{m/s}^2)}{(0.700 \times 10^{-2}\ \text{m})} = 4.00 \times 10^5\ \text{N/m}$$

(b) The frequency of a spring depends on the mass of an object attached to the spring (empty vehicle) and the spring constant and the mass of empty hatchback, *m* = 1,970 kg:

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.00 \times 10^5\ \text{N/m}}{1970\ \text{kg}}} = 2.27\ \text{Hz}$$

Q2 (QID: 4932187). A block connected to a horizontal spring is in simple harmonic motion on a level, frictionless surface, oscillating with amplitude $A$ around $x = 0$. Identify whether each of the following statements is true or false. Note that $x$, $v$, and $a$ represent 1-d vector quantities and can be positive or negative.

(a) If $x = \pm A$, then $|v|=|v_{\max}|$ and $|a|=|a_{\max}|$.

**False**

(b) If $x = 0$, then $|v|=|v_{\max}|$ and $|a|= 0$.

**True**

(c) If $v > 0$, then $a < 0$.

**False**

(d) If $x > 0$, then $a < 0$.

**True**

(e) If $x > 0$, then $v > 0$.

**False**

SOL:

(a) **False**. When $x = \pm A$ (at the maximum amplitude), the block's velocity is $v = 0$.

(b) **True**. At $x = 0$ (at equilibrium), the block has its highest speed ($|v| = |v_{max}|$) and zero acceleration ($||a| = 0$).

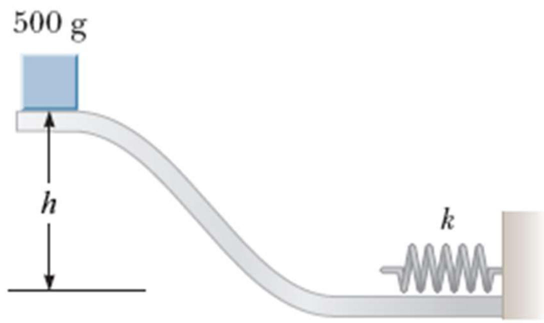
From Hooke's law and Newton's second law,
$\sum F = -kx = ma$, so that $a = 0$ when $x = 0$.

(c) **False**. When $v > 0$, the acceleration $a$ can be positive or negative depending on whether the block is moving toward or away from equilibrium.

(d) **True**. From Hooke's law and Newton's second law, $\sum F = -kx = ma$. That is, the direction of the acceleration is always toward equilibrium (the centre of oscillation).

(e) **False**. When $x > 0$, the velocity $v$ can be positive or negative depending on whether the block is moving toward or away from equilibrium.

Q3 (QID: 4932184). A 500-g block is released from rest and slides down a frictionless track that begins 2.84 m above the horizontal, as shown in the figure below. At the bottom of the track, where the surface is horizontal, the block strikes and sticks to a light spring with a spring constant of 28.0 N/m. Find the maximum distance the spring is compressed.




0.997 m

SOL:

The conservation of mechanical energy from when the block first starts (initial state, *i*) until it comes to rest again (final state, *f*) gives

$$(KE_B + KE_s + PE_B + PE_s)_i = (KE_B + KE_s + PE_B + PE_s)_f$$

where $KE_B$ = Kinetic energy of a block, $PE_B$ = Gravitational potential energy of a block

$KE_s$ = Kinetic energy of a spring, $PE_s$ = Elastic potential energy of a spring

Because the block and the spring are at rest when the block is released (initial state) and the block sticks to the spring at the maximum distance (final state),

$$(KE_B)_i = (KE_s)_i = 0 \text{ and } (KE_B)_f = (KE_s)_f = 0$$

and the spring is at its equilibrium initially and the height of a block is 0 when it strikes a spring,

$$(PE_s)_i = (PE_B)_f = 0$$

Thus, $0 + 0 + mgh_i + 0 = 0 + 0 + 0 + \frac{1}{2}kx_{max}^2$

or $x_{max} = \sqrt{\frac{2mgh_i}{k}} = \sqrt{\frac{2(0.500\text{ kg})(9.80\text{ m/s}^2)(2.84\text{ m})}{28.0\text{ N/m}}} = 0.997$ m

Q4 (QID: 4932180). The position of a 0.300-kg object attached to a spring is described by

$x = (0.210 \text{ m})\cos(0.800\pi t)$. (Assume $t$ is in seconds.)

**Note:** Your calculator must be in *radians mode* for any angle-related calculations in this problem.

(a) Find the amplitude of the motion.

0.210 m

(b) Find the spring constant.

1.89 N/m

(c) Find the position of the object at $t = 0.240$ s.

0.157 m

(d) Find the object's speed at $t = 0.240$ s.

0.352 m/s

SOL:

Comparing the position of an object to the equation of position of an oscillating object,

$$x = A\cos(\omega t) = (0.210 \text{ m})\cos(0.800\pi t)$$

(a) The amplitude, $A = 0.210$ m

(b) From the comparison, the angular frequency, $\omega = 0.800\pi$ rad/s, and the angular frequency depends on the spring constant and the mass of an object:

$$\omega = \sqrt{\frac{k}{m}} \quad \rightarrow \quad k = m\omega^2 = (0.300 \text{ kg})(0.800\pi \text{ rad/s})^2 = 1.89 \text{ N/m}$$

(c) Inserting $t = 0.290$ s,

$$x = (0.210 \text{ m})\cos[(0.800\pi \text{ rad/s})(0.290 \text{ s})] = 0.157 \text{ m}$$

(d) From conservation of mechanical energy, the speed at displacement $x$ is given by,

$$v = \omega\sqrt{A^2 - x^2}$$

For $t = 0.290$ s and $x = 0.157$ m,

$$v = (0.800\pi \text{ rad/s})\sqrt{(0.210 \text{ m})^2 - (0.157 \text{ m})^2} = 0.352 \text{ m/s}$$

Q5 (QID: 4932189). A student enters a tall warehouse and notices a long pendulum that very nearly reaches from the ceiling to the floor. The hanging mass swings back and forth with a period of 20.5 s.

(a) Assuming that the pendulum is nearly as long as the warehouse is tall, what is the warehouse's height (in m)?

104 m

(b) Imagine that the pendulum could be transported to the surface of Mars. If free-fall acceleration there is 3.72 $m/s^2$, with what period (in seconds) will the pendulum swing?

33.3 s

SOL:

(a) The length of the pendulum is almost the same as the height of the museum and the period of a pendulum depends on its length and the acceleration of gravitation.

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \rightarrow \quad L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \mathrm{m/s^2})(20/5\ \mathrm{s})^2}{4\pi^2} = 104\ \mathrm{m}$$

(b) On Mars, the gravitation ($g_M = 3.72\ \mathrm{m/s^2}$is different from that on the Earth.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{104\ \mathrm{m}}{3.72\ \mathrm{m/s^2}}} = 33.3\ \mathrm{s}$$

Q6 (QID: 4932191). A "seconds" pendulum is a pendulum that goes through its equilibrium position once each second. (That is, the period of the pendulum is 2.000 s.) The length of a seconds pendulum is 0.9927 m at Tokyo, Japan and 0.9942 m at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations? (Give your answer to at least 4 decimal places.)

$$\frac{\text{Cambridge}}{\text{Tokyo}} = 1.00151$$

SOL:

The period of the seconds pendulum at both cities should be 2.000 s.

$$T_{\text{Tokyo}} = 2\pi\sqrt{\frac{0.9927\text{ m}}{g_{\text{Tokyo}}}} = 2\pi\sqrt{\frac{0.9942\text{ m}}{g_{\text{Cambridge}}}} = T_{\text{Cambridge}}$$

or

$$\frac{0.9927\text{ m}}{g_{\text{Tokyo}}} = \frac{0.9942\text{ m}}{g_{\text{Cambridge}}} \quad \rightarrow \quad \frac{g_{\text{Cambridge}}}{g_{\text{Tokyo}}} = \frac{0.9942\text{ m}}{0.9927\text{ m}} = 1.00151$$

Q 7 (QID: 4932454). A spring in a toy gun has a spring constant of 8.85 N/m and can be compressed 22.5 cm beyond the equilibrium position. A 1.00-g pellet resting against the spring is propelled forward when the spring is released.

(a) Find the speed of the pellet when it leaves a gun.

21.2 m/s

(b) If the pellet is fired horizontally from a height of 1.10 m above the floor, what is its range?

10.0 m

SOL:

(a) Before the pellet is fired, the energy in initially stored as elastic potential energy in the spring and then it is completely transformed into kinetic energy of the pellet when it leaves the gun (Assume no loss of energy). Thus, the conservation of mechanical energy from when the pellet is fired (initial state, *i*) until it leaves the gun (final state, *f*) gives

$$(KE_S + KE_P + PE_S + PE_P)_i = (KE_S + KE_P + PE_S + PE_P)_f$$

where $KE_S$ = Kinetic energy of a spring, $PE_S$ = Elastic spring potential energy of a spring

$KE_P$ = Kinetic energy of a pellet, $PE_P$ = Gravitational potential energy of a pellet

or, $$0 + 0 + \frac{1}{2}k{x_i}^2 + 0 = 0 + 0 + 0 + \frac{1}{2}m{v_{0h}}^2$$

where $x_i$: Amount of compression of a spring in a toy gun

$v_0$: Horizonal speed of a pellet when it leaves a toy gun

$$\frac{1}{2}k{x_i}^2 = \frac{1}{2}m{v_0}^2 \quad \rightarrow \quad v_0 = \sqrt{\frac{k{x_i}^2}{m}} = \sqrt{\frac{(8.85\ \text{N/m})(0.225\ \text{m})^2}{1.00 \times 10^{-3}\ \text{kg}}} = 21.2\ \text{m/s}$$

(b) Once the pellet leaves at horizonal speed of 21.2 m/s, it starts falling from a height of 1.10 m above the floor due to gravitation. The time required for the pellet to drop 1.10 m to the floor can be determined by the equation:

$$\Delta y = {v_0}_y t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}a_y t^2 \quad \rightarrow \quad t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-1.10\ \text{m})}{(-9.80\ \text{m/s}^2}} = 0.474\ \text{s}$$

Note: ${v_0}_y$ is the vertical speed of a pellet when it leaves a gun. Don't be confused with $v_0$, the horizontal speed of a pellet.

During the time determined above, the pellet moves at the constant speed of $v_0$ in the horizontal direction until it reaches the floor.

Thus, the range (horizonal distance travelled during the flight) is

$$\Delta x = v_0 t = (21.2\ \text{m/s})(0.474\ \text{s}) = 10.0\ \text{m}$$